

TRANSACTION COSTS AND MARKET EFFICIENCY

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Abstract

Previous research suggests that a decline in transactions costs leads to improved economic efficiency. In this paper, we show that such a decline will introduce increasingly uninformed consumers into established markets. Using a model of financial market inefficiency, we show that this increase in uninformed individuals can increase market risk (volatility), can decrease efficiency, and may reduce social welfare even when market participants are perfectly rational. We then test the predictions of our model using data on the retail equities market. Our results suggest that securities that have a large proportion of small trades (presumably disproportionately from small, online retail investors) tend to be less efficient by conventional measures, consistent with our model predictions.

Keywords: eCommerce, transaction costs, market efficiency.

INTRODUCTION

The effects of a decline on transactions costs on the structure and performance of organizations and markets has been a central theme in the information systems literature for many years (Gurbaxani and Whang 1991; Malone et al. 1987). For the most part, research on this topic suggests that lower transactions costs are almost always beneficial. Reductions in transactions costs have been linked to direct cost savings, indirect benefits through improvements in agency costs, monitoring or coordination within existing organizations and markets, and even the creation of new types of market structures that are more efficient (Clemons and Row 1992; Malone and Rockart 1991). A few studies have considered negative effects of reduced transactions costs, especially on intermediaries, but even here the replacement of traditional intermediaries with electronic interaction is socially beneficial, even if it reduces the profitability of existing intermediaries (Clemons and Weber 1994).

However, not all reductions in transactions costs need be beneficial. For instance, classic economic problems such as the “lemons problem” (Akerlof 1973) can be exacerbated if lower transactions costs lead some participants to become more informed than others.¹ Alternatively, reduced transactions costs may introduce new, poorly informed participants into markets that they would otherwise access through a professional intermediary who would screen or temper their information disadvantages. To the extent that market efficiency is affected by the introduction of less uninformed participants, a reduction in transactions costs can impede market efficiency and create real social costs.

¹The classic statement of the lemons problem is when the seller of a used car (a possible “lemon”) knows the value of the car better than a potential buyer, it may be impossible to find an acceptable price for trade. Only when both parties are equally informed (be it completely uninformed or fully informed) can trade actually take place.

While this problem is quite general in any market where a consumer's price is affected by other consumers' information sets,² it is potentially of greatest importance in financial markets, especially in the retail equities markets. For as low as \$5 per trade, anyone can trade stock using a web browser—a cost reduction of as much as a factor of 150 from full service brokerage rates, and perhaps a factor of 30 from traditional discount brokerage rates. With the growth of on-line brokerage (currently at about 29 million accounts—Credit Suisse First Boston 2001) there has been an even greater proliferation of online financial information services ranging from investor chat (e.g., RagingBull.com) to sophisticated analytics for options and derivatives. The combination of plentiful investment information and easy access to trading services has created a “trading boom” among small, retail investors. While more information may make retail investors better informed, retail investors still have a strong information disadvantage relative to market professionals and may show “overconfidence,” causing them to make worse trading decisions (Barber and Odeon 2001). Studies show that investors with access to online trading trade more frequently (Choi et al. 2001) with lower total returns (Barber and Odeon 2001) than investors who lack online access. However, the impact of these investors on market efficiency has not been previously addressed.

Economists have long believed that irrational or less informed investors (“noise traders”) have no impact on asset prices and, therefore, will have no effect on market efficiency. Fama (1965) argued that “sophisticated investors ... will trade against [less informed] ones and drive prices close to intrinsic values.” Moreover, in the course of such trading, the less informed investors will lose money and eventually be weeded from the market. However, these arguments depend critically on the assumption that informed investors can fully arbitrage against the noise traders, which, as Black (1986) pointed out, only holds when the informed investors have unlimited financial resources and infinite time horizons.

In real financial markets, there are at least two factors that limit informed investors' arbitrage behavior (see DeLong et al. 1990; Lee et al. 1991). First, many market professionals are measured and compensated on short run performance in managing a finite portfolio. As a result, they may be unwilling to engage in arbitrage requiring significant time or open-ended financial commitments.³ For instance, at the peak of the stock market a few years ago, CNBC interviewed a number of experienced mutual fund managers. Almost all of them agreed that eCommerce stocks were significantly over-valued, but none of them were willing to short sell these stocks for fear that they could rise further for a long period before falling back to the intrinsic value. For managers measured by quarterly performance, the risks were simply too high to justify the arbitrage. Second, because short-selling shares is subject to different costs, margin requirements, and regulatory restrictions than taking long positions (buying shares), this deters managers from taking short positions even when they believe shares are overvalued.

Based on these intuitions, DeLong et al. (1990) developed a model demonstrating that “arbitrage does not eliminate the effect of noise *because noise itself creates risk* [under finite time horizons]” (emphasis in original). As a result, the assets become less attractive to informed investors and prices become lower. Thus, both noise traders and informed investors will expect higher returns and higher risks (volatility) in this market.

We consider the effects of declining transactions costs on the performance of financial and other information intensive markets. In the first part of the paper, we develop a simple model that links declining transactions costs to an increase in direct market participation by less informed investors (who no longer have intermediaries to compensate for their information disadvantages). This model also shows a decline in margins for intermediaries. While this is intuitive, the model is interesting because it generates these results with few assumptions. Then, building on the model developed by DeLong et al., we consider the problem that an increase in participation by uninformed investors creates on overall market efficiency. This model generates a specific set of testable predictions that we examine using data on the price behavior of recent initial public offerings (IPOs), a set of stocks that is particularly favored by online traders. While our empirical measures of various model constructs are far from perfect, we do find evidence that stocks heavily traded by in small lots (presumably by retail investors, especially those online) tend to show greater violations of standard indicators of market efficiency.

Our work is related to research on market price variance and flows of market information (Barclay et al. 1990; Charles et al. 1994), but differs in that we consider the impact of changing the balance of informed and uninformed investors rather than the effects of the arrival of new information. It differs from classical transaction cost approaches in that it incorporates the issue of market efficiency in addition to market structure. Our work is most closely related to research on market inefficiency, although we propose a specific process by which uninformed investors are introduced into the market.

²In addition to financial markets, these concepts apply generally to auction markets, markets with imperfect price discrimination mechanisms, or those with participation externalities.

³Detailed discussions can be found in DeLong et al. (1990) and in Lee et al. (1991).

MODEL

Model: Intermediary Fees and Investor Participation

We consider a market with products of uncertain values with two types of market participants: individuals and a monopolistic⁴ professional agent. An individual can either access the market directly (incurring a transaction cost) or can use the agent by paying a fixed fee. Agents are risk neutral, face zero transactions costs, and have perfect information regarding the value of the underlying good (a probability distribution). Individuals are risk-averse, face positive transaction costs, and have worse information regarding valuation of the available assets than the agent (in the form of a mean-preserving spread of the true distribution).

Formally, we make the following assumptions:

1. The true value V of the product is normally distributed with mean θ and variance σ^2_θ . For individual i , a noise of $\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$ is mixed in the perceived information of the value. The exact value of σ^2_ε is private information to the individual (i), while the distribution of σ^2_ε , $F(\sigma^2)$, is known to the agent. The hazard function⁵ of $F(\sigma^2)$ is monotonically increasing.
2. Each individual's utility is a constant absolute risk aversion (CARA) function:

$$U = -e^{-(2\gamma)w}$$

where γ is the coefficient of absolute risk aversion. Without loss of generality, γ is set to 1 for all individuals, since σ^2_ε alone is sufficient to represent individual differences.⁶

3. The individual incurs a transaction cost of C_i if accessing the market directly, and pays a fee to the agent of C_f if using the agent (but no additional transaction costs).

The expected utility of each choice is calculated as follows:

Choice 1: No purchase.

$$E(U_1) = 0;$$

Choice 2: Purchase directly on the market:

$$U_2 = V + \varepsilon_i - C_i; \text{ where } V \sim (\theta, \sigma^2_\theta), \varepsilon_i \sim (0, \sigma^2_\varepsilon), \text{ thus} \\ E(U_2) = \theta - \sigma^2_\varepsilon - \sigma^2_\theta - C_i$$

Choice 3: Purchase through the agent:

$$U_3 = V - C_f; \text{ where } V \sim (\theta, \sigma^2_\theta), \text{ thus} \\ E(U_3) = \theta - \sigma^2_\theta - C_f$$

Individuals choose the transaction medium that provides the highest expected utility. We first consider the agents participation constraint and then characterize optimal pricing by the agent.

⁴While the model is built for a monopolistic agent, results are qualitatively similar as long as competition is imperfect.

⁵The hazard function of distribution $F(\theta)$ is: $f(\theta)/[1 - F(\theta)]$.

⁶This is because under CARA utility and normally distributed wealth, expected utility has the form $E(U(w)) = E(e^{-2\gamma w}) = \mu_w - \gamma \sigma_w^2$ where the mean and variance of w are given by (μ_w, σ_w^2) .

Proposition 1: If $\theta > \sigma_\theta^2$, all individuals will participate in the market (either via the agent or directly), otherwise no individuals will participate.

Proof: See Appendix.

As a consequence, we restrict ourselves to conditions where individuals do purchase and the problem reduces to a choice of channel (direct vs. intermediated). Proposition 2 provides the optimal pricing choice for the agent.

Proposition 2: There is a unique solution of C_t^* defined by $C_f^* = H^{-1}(C_f^* - C_t)$ that maximizes the agent's profit.

Proof: See Appendix.

With these results, it is straightforward to compute the comparative statistics of agent pricing as transactions cost change:

Proposition 3: When individuals' direct transaction cost decreases, the agent reduces its fee C_t but the price difference between the transaction cost and the agent's fee increases.

Proof: See Appendix.

These results collectively suggest that as transactions costs decline, the agent receives less revenue and a smaller share of transactions, even when agents have a cost advantage in accessing the market and better information than individuals. Using intermediate results from the previous proofs, we can also examine how participating in these markets is affected by transactions costs. We find that as transactions costs decline, individuals increase their use of the market, which results in an increase in overall degree of "uninformedness" (defined as the average amount of noise variance in their estimates of value) of the individuals that access the markets directly.

Corollary 3.1: When the individual's direct transaction cost decreases, less informed individuals enter into the market directly. The average "uninformedness" of all direct participants increases.

Proof: See Appendix.

The latter results hold as long as individuals' assessment of information is not substantially improved by direct market access or improved concurrently with direct market access due to other factors. Empirical research suggests that individual trading behavior is consistent with this assertion. It has been shown that research activity by retail investors does not increase investment performance (Barber and Odean 2001; Choi et al. 2001).

Our results, therefore, suggest that declining transactions costs will be associated with an overall reduction in the informedness of traders in the market. In the next section, we consider the implications of this result for market efficiency. For tractability, we restrict individuals to be one of two types: one with better information or lower variance (σ_L^2), the other with higher variance (σ_H^2) where $\sigma_L^2 < \sigma_H^2$. Let the proportion of individuals with better information be represented by γ .

Corollary 3.2: A profit maximizing agent will serve both types of individuals when $\gamma > 1 - \frac{C_t + \sigma_L^2}{C_t + \sigma_H^2}$, and only serve the less informed agents otherwise.

Proof: See Appendix.

As before, as transactions costs decline sufficiently, the market shifts from full intermediation to having the better informed customers accessing the market directly. This corollary is a special case of the previous one under discrete types of individual investors. This simplification allows us to consider direct access by only a single type of consumer in our analysis of market efficiency impact.

Model: Market Efficiency

To examine the impact of increased individual participation on efficiency we consider a variant of the De Long, Shleifer, Summers, and Waldmann (DSSW) model. DSSW considers an overlapping generations setting with an infinite time horizon subdivided into periods.⁷ There are two types of individuals, D's who access the market directly and A's who utilize agents. Both types of individuals live for two periods. Each period includes both "old" individuals (those from the previous period) and "young" individuals just entering the market, but the total number of customers is constant over time. For young individuals, a proportion α are D's and $1-\alpha$ are A's. For simplicity, we assume that the total number of participants does not change over time.

There are two assets with identical dividends (r): a safe asset, s , available in infinite supply, and a risky asset, u , which has a fixed supply and therefore a market price that can vary based on supply and demand. Individuals make holding decisions when they are young to maximize expected utility when they are old. If the market were perfectly efficient, the price of the safe and risky assets would be the same (representing their dividend rates) and there would be no price volatility. However, the presence of uninformed individuals will alter these equilibrium prices.

To explicitly model the imperfect information of the individuals, we assume that in each period there is a system-wide noise $\rho_t \sim N(0, \sigma_L^2)$ ⁸ on the estimation of expected price of the risky assets. The A's are able to screen out this noise while the D's cannot. We can then examine the impact of declining transactions costs on the price variance of the risky asset.

Proposition 4: As transactions costs decrease, market prices become more volatile.

Proof: See Appendix.

While the proof is somewhat involved, the intuition is simple. As transactions costs decline, more individuals enter the market directly. The presence of these imperfectly informed investors creates fluctuation in the demand and supply for the risky asset and, therefore, price fluctuations. This result can be generalized to multiple type investors.

Because we know the behavior of prices, we can predict analytically the behavior of standard tests of market efficiency. To test market efficiency, it is common to calculate the first-order autocorrelation coefficient of prices. The weak form of the efficient market hypothesis implies that this autocorrelation should be zero (past prices should not predict future prices). For our model, the first-order autocorrelation is given by:

$$Cor(\Delta p_{t+1}, \Delta p_t) = \frac{B(F(C_t))^2}{(1+r)^2} Cor(\Delta \rho_{t+1}, \Delta \rho_t) \quad (1)$$

Market autocorrelation is thus a function of the rate of return, the transaction cost (the function B is defined in the proof of Proposition 4), and the autocorrelation of system-wide noise (investor sentiment or other systematic forecast errors). While we cannot measure these factors directly, previous literature has extensively documented the existence of this correlation. Given that this correlation is nonzero, we can derive the relationship between declining transaction costs and this measure of market efficiency:

$$\frac{dCor(\Delta p_{t+1}, \Delta p_t)}{dC_t} = \frac{2B(F(C_t))}{(1+r)^2} \frac{dB(F(C_t))}{dC_t} Cor(\Delta \rho_{t+1}, \Delta \rho_t)$$

⁷Our model differs from DSSW in that individuals are aware that their estimates are noisy. Relaxing this assumption strengthens the result. Moreover, we incorporate two types of investors as well as transactions costs, which are not present in the original DSSW model.

⁸This corresponds to the situation where individuals with noise σ_L^2 enter the market directly.

Then, we can easily show that any autocorrelation in system-wide noise will be exacerbated by declines in transactions costs (and the direct participation of investors with imperfect information).⁹

Social Welfare

Given that the “quality” of asset prices deteriorate as more investors with imperfect information enter the market directly, it is useful to consider how this deterioration affects overall social welfare, defined as aggregation of all individual utilities and the utility of the agent. While we have already seen that agent profits decline as transaction costs decline, this could arise because of redistribution to individuals as well as efficiency losses. However, from a policy perspective we are most concerned about total welfare. While most previous research takes it for granted that lower transaction costs improve social welfare, once again this may not hold in the type of scenarios we are considering.

Proposition 5: Declines in transactions costs decrease total social welfare.

Proof: See Appendix.

We calculate the total social welfare in this market by adding the utilities of individuals using the agent, the utilities of individuals bypassing the agent, and the surplus of the agent. It can then be further decomposed into (1) value of transaction, (2) total transaction costs, (3) costs of imperfect information (noise), and (4) costs of uncertainty in assets returns. Clearly, in a market with two types of investors, the lower transaction cost will not change the valuation of transaction, but the total transaction costs, costs of noise and costs of uncertainty all increase as a result. Thus the social welfare decreases.

EMPIRICAL TEST: THE GROWTH IN ON-LINE BROKERAGE AS A NATURAL EXPERIMENT

Empirical Model

To test our model, we consider the online brokerage industry. The rise of on-line trading dramatically reduces the direct transaction cost and provides a largely exogenous but measurable increase in the number of traders in the market, which previous research has suggested may be less informed. One might expect this effect to be largest in newly issued Internet stocks, favored by online retail investors.

Taking logarithms of our earlier model of first order autocorrelation (equation 1) and applying first-degree approximation on B yields the following estimating equation:

$$\log | \text{Cor}(\Delta p_t, \Delta p_{t+1}) | = a + b \log F(\sigma^2(C_t)) + \log | \text{Cor}(\Delta \rho_t, \Delta \rho_{t+1}) |$$

We are unable to test this equation directly due to the lack of available data on the correlation of system-wide noise. However, we can provide related evidence on the relationship between investor participation and stock price efficiency.

Notice that $F(\sigma^2(C_t))$ is just the percentage of individuals who trade directly on the market. The remaining participants either trade through the help of financial professionals or simply invest in mutual funds managed by professionals. To estimate this quantity, we hypothesize that the percentage of individuals trading directly on the market is highly correlated to the percentage of small trades, namely the percentage of trade of value less than \$10,000. We use TAQ data to compute these fractions based on intra-day trade-by-trade transaction data for all securities listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the Nasdaq National Market System (NMS) and SmallCap issues. Thus, we can estimate the small investor participation as:

⁹ $\frac{2B(F(C_t))}{(1+r)^2} \frac{dB(F(C_t))}{dC_t} = \frac{2B(F(C_t))\sigma_p^2}{(1+r)^2} \frac{dB(F(C_t))}{d\sigma^{2*}} \frac{d\sigma^{2*}}{dC_t} < 0$, since $\frac{d\sigma^{2*}}{dC_t} < 0$.

$$\text{Proxy of } F(\sigma^2(C_i)) = \frac{\text{Total Value of Trades } < \$10,000}{\text{Total Value of Trades}}$$

General Test of Market Efficiency

Our dataset comprises of 3,126 IPOs from July 1, 1995, to March 31, 2000, excluding REITs and ADRs. For each stock in the list, we first calculate N_i the percentage of the value of trades less than \$10,000 using the TAQ data.

Because individual stock returns have a high degree of random noise, we follow the standard procedure by forming portfolios to average out the idiosyncratic noise. We created 10 equal-size, equal-weighted portfolios based on the proportion of small trades (N_i). To improve the power of the statistical test, we use the Q-statistic proposed by Box and Pierce for the test of market inefficiency.

$$Q_m = T(T + 2) \sum_{k=1}^m \frac{\rho^2(k)}{T - k},$$

where $\rho(k)$ is the k^{th} lag autocorrelation of the portfolio returns. T is number of observations.

The efficient market hypothesis indicates that the first-differences of stock returns are uncorrelated at all leads and lags. Under this null hypothesis, Q_m is a chi-square distribution with m degrees of freedom, where m is the number of lags.

For each of the 10 portfolios, we calculate the Q-statistics for up to five lags. The result is clear (see Figure 1). The Q-statistics for the four most directly traded portfolios are significant at the 5% level, where the rest of the portfolios are not significant at all. Thus, while we are unable to directly test all implications of our model, the general prediction that market efficiency is negatively affected by uninformed investors is consistent with our model predictions.

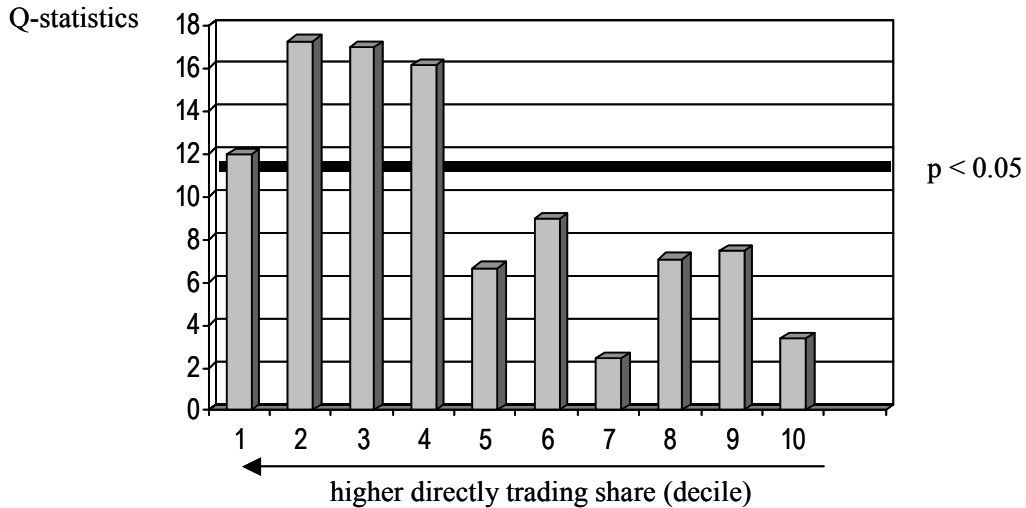


Figure 1. Autocorrelation Tests

CONCLUSIONS

In this paper, we formulate a set of models that relate a decline in transactions costs to increased participation in markets by uninformed investors, show theoretically that this increase in participation can adversely affect market efficiency, and demonstrate

that a simple empirical test is consistent with these models. While we are unable to perform a complete empirical test with available data, this initial test is broadly consistent with our model predictions.

Our model is one of the few analyses that consider how a decline in transactions costs can have adverse effects. Similar effects are likely to appear in markets that are believed to be less efficient than securities markets, thus suggesting that these types of concerns may have wide ranging effects. For instance, online auctions, business to business (B2B) exchanges, and other types of new trading mechanisms might be especially vulnerable to these effects, especially those previously managed by well-informed intermediaries or where transactions costs declines have increased the number of market participants. In future research, we hope to extend this model to other contexts beyond financial markets as well as developing measures that enable the full structural form of our model predictions to be tested directly.

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APPENDIX

Proof of Proposition 1: Consider the intermediary fee C_f . If $C_f > \theta - \sigma_\theta^2$, we have $E(U_3) = \theta - \sigma_\theta^2 - C_f < 0$. As a result, no individual will use the agent. It is, therefore, a dominated strategy for the agent. Since a profit-maximizing agent will always choose $C_f < \theta - \sigma_\theta^2$, which implies $E(U_e) > 0$, Choice 1 is dominated by Choice 3 for all individuals. On the other hand, if $\theta < \sigma_\theta^2$, the benefit of the transaction is always less than the cost of the uncertainty of its benefit and no individuals will participate in the market. **Q. E. D.**

Proof of Proposition 2: We first consider the individual's choices. From Proposition 1, the individual only needs to make a choice between purchasing directly on the market and through the agent. For the marginal individual i^* , who is indifferent between two choices, we have $\sigma_i^2 = \sigma^{2*} = C_f - C_t$. All individuals with $\sigma_i^2 > C_f - C_t$ will use the agent for the transaction, while the rest will make the purchase directly in the market.

Second, we consider the choice of the agent. The agent always chooses an optimal fee C_f to maximize its total expected profit: $E(\Pi) = C_f [1 - F(C_f - C_t)]$, where F is the distribution function of σ^2

Define the hazard function $H(\sigma^2) = f(\sigma^2) / [1 - F(\sigma^2)]$. Since $H(\sigma^2)$ is monotonically increasing (by assumption),¹⁰ we get the unique solution for C_f^* implicitly identified in the following equations.

$$\begin{aligned} C_f^* &= H^{-1}(C_f^* - C_t) & \text{if } f^{-1}(0) > C_t \\ C_f^* &= C_t & \text{if } f^{-1}(0) < C_t \end{aligned}$$

For the rest of the paper, we only consider the cases where $f^{-1}(0) > C_t$. **Q. E. D.**

Proof of Proposition 3: Apply the implicit function theorem to $C_f^* = H^{-1}(C_f^* - C_t)$.

Let $g(C_f^*, C_t) = C_f^* - H^{-1}(C_f^* - C_t)$.

$$\frac{dC_f^*}{dC_t} = - \frac{\partial g / \partial C_t}{\partial g / \partial C_f^*} = - \frac{H^{-1}(C_f^* - C_t)}{1 - H^{-1}(C_f^* - C_t)} > 0, \text{ since } H(\sigma^2) \text{ is monotonically increasing.}$$

That is, when individuals' direct transaction cost decreases, the agent reduces its fee C_f^* .

Let D be the difference between the agent fee and the direct transaction cost, $C_f^* - C_t$. Define

$$h(D, C_t) = C_f^* - H^{-1}(C_f^* - C_t) = D + C_t - H^{-1}(D).$$

¹⁰Most commonly used distribution functions satisfy this assumption including the normal, exponential and gamma, beta distributions.

$$\frac{dD}{dC_t} = -\frac{\partial h/\partial C_t}{\partial h/\partial D} = -\frac{1}{1-H^{-1}(D)} < 0, \text{ since } H(\sigma^2) \text{ is monotonically increasing.}$$

That is, when individuals' direct transaction cost decreases, the price difference between the direct transaction cost C_t and the agent fee C_f^* increases. **Q. E. D.**

Proof of Corollary 3.1: As stated in proposition 1, the margin individual i has

$$\sigma^{2^*} \equiv C_f - C_t = D$$

Apply Proposition 3, $\frac{d\sigma^{2^*}}{dC_t} = \frac{dD}{dC_t} < 0$, i.e., σ^{2^*} is increasing when the direct transaction cost decreases.

Define "uninformedness" S as the average excess variance of the individuals who participate in the market directly. That is,

$$S = \int_0^{\sigma^{2^*}} xf(x)dx$$

Applying Leibnitz's Rule, we have

$$\frac{dS}{dC_t} = \sigma^{2^*} f(\sigma^{2^*}) \frac{d\sigma^{2^*}}{dC_t} < 0$$

That is, "do-it-yourself" individuals become less informed as an aggregate, when the direct transaction cost decreases. **Q. E. D.**

Proof of Corollary 3.2: Bypassing the agent becomes desirable when $C_f > C_t + \sigma_i^2$. If the agent charges $C_f > C_t + \sigma_H^2$, no one will use its service. If the service charge is between $C_t + \sigma_L^2$ and $C_t + \sigma_H^2$, only the less informed individual will use its service. When the charge falls below $C_t + \sigma_L^2$, everyone will use the agent. The agent is expected to receive a total revenue of $(1-\alpha)(C_t + \sigma_H^2)$ by charging $C_t + \sigma_H^2$, or $C_t + \sigma_L^2$ by charging $C_t + \sigma_L^2$. Thus, the threshold value for γ is:

$$1 - \frac{C_t + \sigma_L^2}{C_t + \sigma_H^2}. \quad \mathbf{Q. E. D.}$$

Proof of Proposition 4: Again, all participants have a constant absolute risk aversion function:

$U = -e^{-2\gamma W}$, where γ is assumed to be 1, without loss of generality.

The A 's choose the amount λ_t^i of the risky asset u to maximize expected utility:

$$E(U) = \lambda_t^A [r + {}_t p_{t+1} - p_t(1+r)] - (\lambda_t^A)^2 ({}_t \sigma_{p_{t+1}}^2) - C_f$$

where ${}_t p_{t+1}$ is the one-period-ahead expectation of price at period $t+1$; ${}_t \sigma_{p_{t+1}}^2$ is the one-period-ahead expectation of the variance of price at period $t+1$.

Similarly, the D 's chooses the amount λ_t^D to maximize expected utility:

$$E(U) = \lambda_t^D [r + {}_t p_{t+1} - p_t(1+r) + \rho_t] - (\lambda_t^D)^2 ({}_t \sigma_{p_{t+1}}^2 + \sigma_L^2) - C_t$$

Solving the above two equations, we have the participants' demand for unsafe asset u :

$$\lambda_t^A = \frac{r + {}_t p_{t+1} - p_t(1+r)}{2({}_t \sigma_{p_{t+1}}^2)}$$

$$\lambda_t^D = \frac{r + {}_t p_{t+1} - p_t(1+r) + \rho_t}{2({}_t \sigma_{p_{t+1}}^2 + \sigma_L^2)}$$

Both A 's and D 's must sell all the assets acquired in the earlier period. Therefore, the demands of the first period generation must sum to one in equilibrium. That is:

$$\lambda_t^D F(\sigma^{2*}) + [1 - F(\sigma^{2*})] \lambda_t^A = 1$$

p_t is, therefore, implicitly defined by the system-wide noise (ρ_t), the risk-free risk (r), the direct transaction cost (C), the distribution of information process ability of individuals, and of moments of the one-period-ahead distribution of p_{t+1} .

$$F(\sigma^{2*}) \frac{r + {}_t p_{t+1} - p_t(1+r) + \rho_t}{2({}_t \sigma_{p_{t+1}}^2 + \sigma_L^2)} + [1 - F(\sigma^{2*})] \frac{r + {}_t p_{t+1} - p_t(1+r)}{2({}_t \sigma_{p_{t+1}}^2)} = 1$$

A little algebra reveals that:

$$r + {}_t p_{t+1} - p_t(1+r) = \frac{{}_t \sigma_{p_{t+1}}^2 [2({}_t \sigma_{p_{t+1}}^2 + \sigma_L^2)] - F(\sigma^{2*}) {}_t \sigma_{p_{t+1}}^2 \rho_t}{{}_t \sigma_{p_{t+1}}^2 + \sigma_L^2 - F(\sigma^{2*}) \sigma_L^2} = A - B \rho_t$$

where

$$A = \frac{{}_t \sigma_{p_{t+1}}^2 [2({}_t \sigma_{p_{t+1}}^2 + \sigma_L^2)]}{{}_t \sigma_{p_{t+1}}^2 + [1 - F(\sigma^{2*})] \sigma_L^2}$$

$$B = \frac{F(\sigma^{2*}) {}_t \sigma_{p_{t+1}}^2}{{}_t \sigma_{p_{t+1}}^2 + [1 - F(\sigma^{2*})] \sigma_L^2}$$

Thus,

$$p_t = \frac{1}{1+r} [r + {}_t p_{t+1} - A + B\rho_t]$$

In equilibrium, this equation must hold in all periods. We can thus solve for p_t recursively:

$$p_t = 1 + \frac{B\rho_t}{1+r} - \frac{A}{r}$$

Clearly, in the above equation, only ρ_t (the system-wide noise) is a random variable. We can then calculate the one-period-ahead variance ${}_t \sigma_{p_{t+1}}^2$ as:

$${}_t \sigma_{p_{t+1}}^2 = \sigma_{p_{t+1}}^2 = \frac{B^2 \sigma_L^2}{(1+r)^2}$$

Differentiate the expression for variance and apply the chain rule:

$$\frac{d\sigma_{p_{t+1}}^2}{dC_t} = \frac{2B\sigma_L^2}{(1+r)^2} \frac{dB}{dC_t} = \frac{2B\sigma_L^2}{(1+r)^2} \frac{dB}{d\sigma^{2*}} \frac{d\sigma^{2*}}{dC_t} < 0 \quad (\text{Corollary 3.1})$$

Therefore, the variance of price is increasing as a result of lower transaction cost. Notice that the variance is purely generated by the presence of less informed individuals affected by the system-wide noise. **Q. E. D.**

Proof of Proposition 5: The total social welfare is the aggregation of utilities of individuals accessing the market directly, utilities of individuals going through the agent and the utility of the agent. As we have shown in Corollary 3.2, after the transaction drops, the σ_L^2 will enter the market directly, while the σ_H^2 stay with the agent. Therefore, the total social welfare before the change in transaction costs is simply:

$$W(C_t^H) = \theta$$

The social welfare after the change is:

$$W(C_t^L) = \theta - \sigma_L^2 - \sigma_\theta^2 - \gamma C_t - \frac{B^2 \sigma_L^2}{(1+r)^2} < W(C_t^H)$$

The social welfare decreases as a result. **Q. E. D.**